Finance I – Econ 233

Data Case 1

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1. The annualized standard deviation for the S&P 500 is 29.11% when daily data is used. On the other hand, the annualized standard deviation for the S&P 500 is 21.01% when monthly data is used. The difference is large, and reflects the fact that we have less information when using monthly data.

With daily data, the annualized arithmetic return for the S&P 500 is 3.40% and the annualized geometric return is **-**0.82%. With monthly data, the annualized arithmetic return for the S&P 500 is 3.04% and the annualized geometric return is **+**0.82%.

2. The optimal risky-portfolio using the Markowitz model and excess return is:

**NO shorting** **YES shorting**

S&P 500: 69.93% 79.42%

Google: 3.99% 7.65%

Apple: 0.00% -10.58%

Exxon Mobile: 16.57% 14.61%

Walmart: 9.51% 8.90%

Sharpe Ratio: 0.1783 0.1794

We note that all of our assets have a high positive correlation with the S&P 500. Thus, we will invest mostly in the S&P 500 because it has the highest sharpe ratio. If we allow short selling we see that we short Apple. This is because Google dominates Apple and they both are equally correlated with the S&P 500.

3. The optimal risky-portfolio using the index model and the total return:

**NO shorting** **YES shorting**

S&P 500: 70.81% 80.55%

Google: 2.67% 2.69%

Apple: 0.00% -9.91%

Exxon Mobile: 16.28% 16.37%

Walmart: 10.24% 10.30%

In order to do this we needed: (1) the beta of each asset, which we obtained using a linear regression of each asset with the S&P 500 and (2) the firm-specific risk, which we obtained by squaring the standard error of the residuals for each asset. Then, using the observed variance of the market (S&P 500) and the above two values, we calculated the covariance matrix.

4. Calculated variance (e.g. Google: 15.03% annualized) is an approximation of observed variance (e.g. Google: 15.02% annualized) and stems from assumptions about returns provided by the single factor model.

5/6. The optimal risky-portfolio using the Treynor-Black model and excess returns is:

**(α≠0)** **(α=0)**

S&P 500: 97.20% 100%

Google: -0.27% 0.00%

Apple: 1.51% 0.00%

Exxon Mobile: 0.26% 0.00%

Walmart: 1.30% 0.00%

In this model short selling and buying on margin is dictated by the value of alpha. Thus, the only asset we short sell is Google since it has a slightly negative alpha value.

When the alpha values are all zero the optimal risky-portfolio becomes the market (S&P 500).

7. We see that the expected returns, standard deviations, and Sharpe ratios across the first three portfolios are all very similar.

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| --- | --- | --- | --- | --- | --- | --- |
|  | Expected Returns | S&P500 | Google | Apple | Exxon Mobil | Walmart |
| Markowitz | 5.23% | 79.42% | 7.65% | -10.58% | 14.61% | 8.90% |
| Index | 5.22% | 80.55% | 2.69% | -9.91% | 16.37% | 10.30% |
| TB (α≠0) | 5.10% | 97.20% | -0.27% | 1.51% | 0.26% | 1.30% |
| TB (α=0) | 5.14% | 100% | 0% | 0% | 0% | 0% |

|  |  |  |
| --- | --- | --- |
|  | Standard Deviation | Sharpe Ratio |
| S&P500 | 29.11% | 0.1766 |
| Google | 38.76% | 0.1233 |
| Apple | 40.70% | 0.1051 |
| Exxon Mobil | 33.84% | 0.1560 |
| Walmart | 23.84% | 0.1166 |
| Markowitz | 27.95% | 0.1794 |
| Index | 27.86% | 0.1793 |
| TB (α≠0) | 28.90% | 0.1766 |
| TB (α=0) | 29.91% | 0.1766 |

We don’t report each security’s Sharpe ratio across the 4 model because they are very close.

8. We allocate 21.45% of our investment into the optimal risky-portfolio and the other 78.5% into treasury bills. Given the $5 million we have to invest, these weights translate into $1,072,706 in our optimal risky-portfolio and $3,927,294 in treasury bills. This is because our coefficient of risk aversion is high and positive (A=3), which implies that we are risk averse.

9. Here we are assuming that the returns in our complete portfolio using the index model follow a normal distribution. We have an annualized expected total return of 1.29% and a standard deviation of 5.98% on the complete portfolio. The probability that we will have a negative total return over a one year period is 41.44%